

*Electronic version of an article published as [Operations Research Letters, 2002, vol. 30, No. 1, p. 69-70] [DOI: [http://dx.doi.org/10.1016/S0167-6377\(02\)00105-0](http://dx.doi.org/10.1016/S0167-6377(02)00105-0)]
© [copyright Elsevier]*

A remark about “A comment on consecutive-2-out-of- n systems”

Albert Corominas

Institute of Industrial and Control Engineering, Universitat Politècnica de Catalunya, Av. Diagonal 647, 08028 Barcelona, Spain

Abstract

Deĭneko and Woeginger (Oper. Res. Lett. 28 (2001) 169) present a proof that a result of Du and Hwang (Math. Oper. Res. 11 (1986) 187) about the optimum arrangement of the items in a consecutive-2-out-of- n cycle system is a simple special case of Supnick's result about the optimum solution of the travelling salesman problem with certain specially structured distance matrices. In this paper, it is pointed out that Deĭneko and Woeginger's proof contains a flaw that makes its conclusion invalid.

Author Keywords: Reliability; 2-out-of- n system; Supnick conditions; Probability

1. Consecutive-out-of- n cycle systems and Supnick's matrices for the TSP

Deĭneko and Woeginger [1] study the relation between two apparently unconnected results. The first one, owed to Du and Hwang [2] is about the optimal ordering of the items in a consecutive-2-out-of- n cycle system (i.e., a system, with n items ordered into a cycle, that fails if and only if two consecutive items both fail). The second, from Supnick [3] is on the shortest and longest hamiltonian tours in graphs whose distance matrices fulfil certain specific conditions.

As far as the consecutive-2-out-of- n cycle system problem is concerned, it is assumed, without loss of generality, that—being p_i ($i=1, \dots, n$) the probabilities that the items of the system work— $p_i \leq p_{i+1}$ ($i=1, \dots, n-1$). In [1 and 2] it is also assumed that the items are stochastically independent.

2. About the proof of the connection between both problems

In [1] a proof is presented in order to show that Du and Hwang's result is a special case of Supnick's.

The authors prove that the expression

$$\begin{aligned} & (p_{\pi(1)} + p_{\pi(n)} - p_{\pi(1)}p_{\pi(n)}) \prod_{i=1}^{n-1} (p_{\pi(i)} + p_{\pi(i+1)} \\ & - p_{\pi(i)}p_{\pi(i+1)}) \end{aligned} \quad (1)$$

takes its maximum at $\pi=\sigma^*$, where σ^* corresponds to the optimal permutation of the items, according to Du and Hwang's result.

Then it is claimed that finding an arrangement that maximises the probability that the whole system works corresponds to finding a permutation that maximises the product in Eq. (1), since each factor of the product corresponds to a pair of consecutive items and gives the probability that at least one of them works.

However, in spite of being true that the factors of the product are the aforementioned probabilities, the product is not the probability that the whole system works, since the probabilities corresponding to two consecutive pairs are not independent, because of the pairs share a common item. Therefore, the equivalence between maximising the product in Eq. (1) and maximising the probability that the whole system works should be shown.

This may be clarified with a simple example. Let us assume $n=3$ and $p_i=p$ ($i=1,2,3$). Obviously, in order that the system works at least two items must work and the probability of this event is $p^3+3p^2(1-p)=3p^2-2p^3$; instead, the expression in [1] gives $(2p-p^2)^3$, which is different.

The conclusion, therefore, is that the proof presented in [1] does not allow establishing the connection between the results in [2 and 3] that has been supposed.

References

1. V.G. Deĭneko and G.J. Woeginger , A comment on consecutive-2-out-of- n systems. *Oper. Res. Lett.* 28 (2001), pp. 169–171.
2. D.Z. Du and F.K. Hwang , Optimal consecutive-2-out-of- n systems. *Math. Oper. Res.* 11 (1986), pp. 187–191.
3. F. Supnick , Extreme Hamiltonian lines. *Ann. Math.* 66 (1957), pp. 179–201.